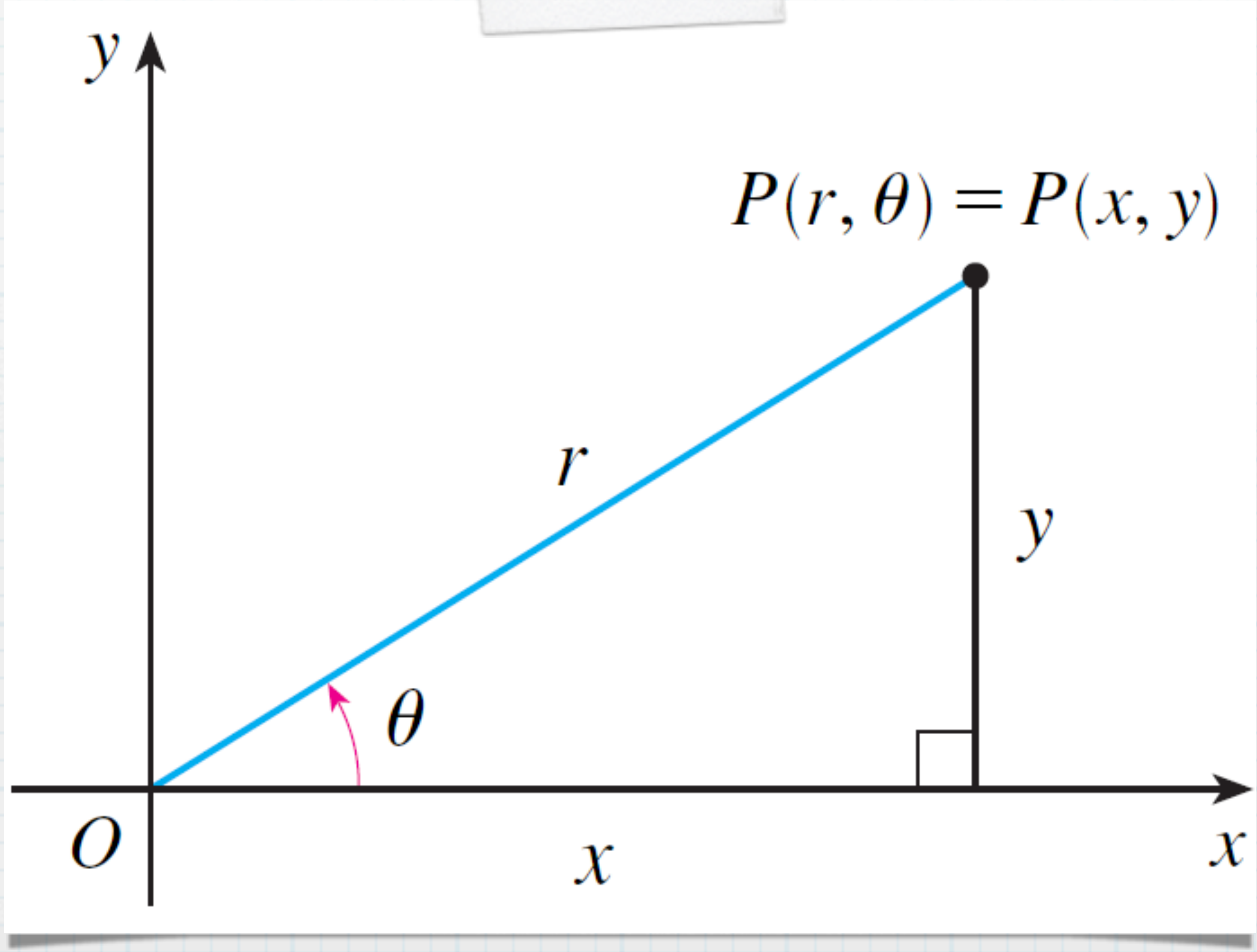
Section 9.7

Cylindrical and Spherical Coordinates

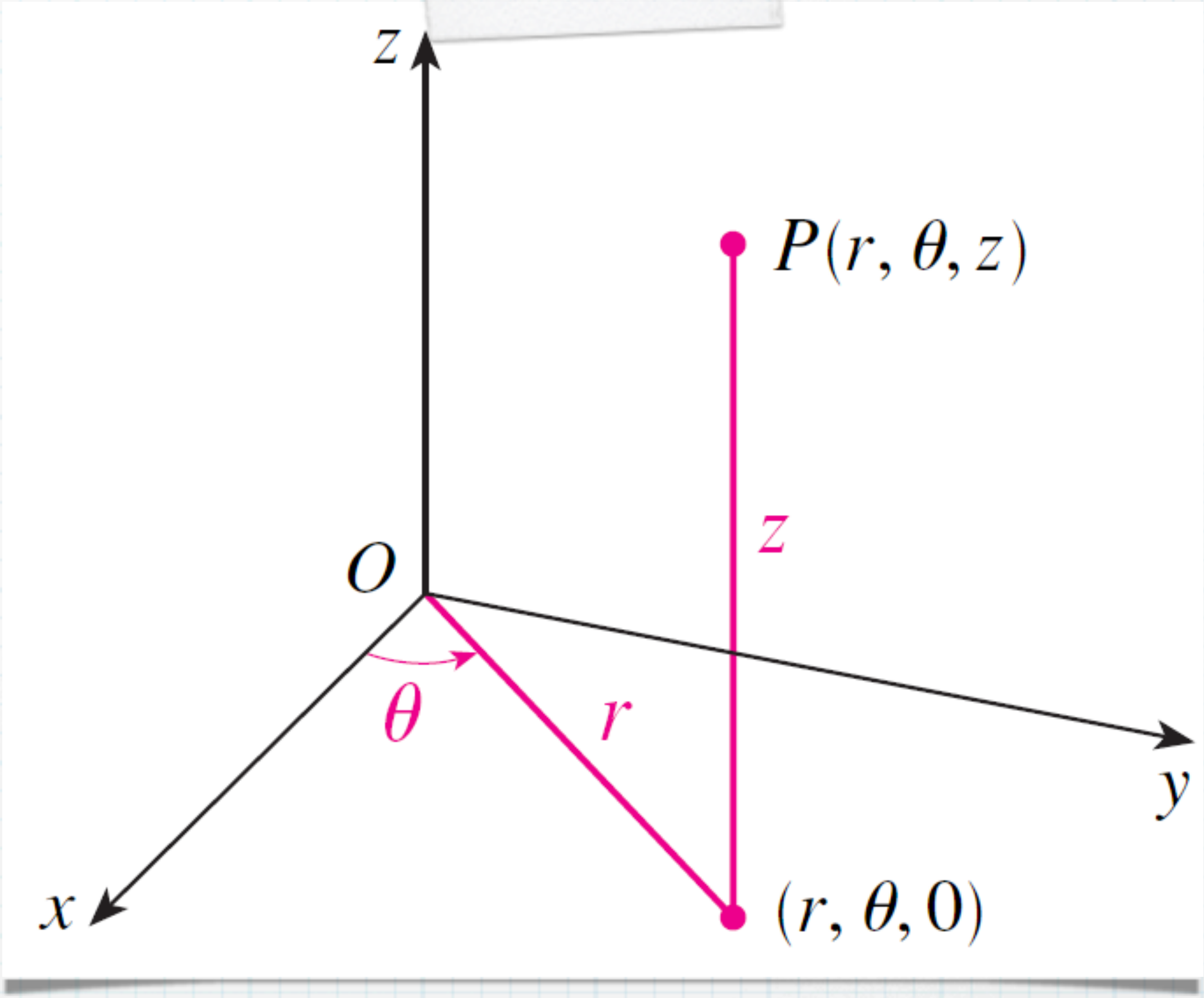
Polar Coordinates

$$x = r\cos\theta \qquad y = r\sin\theta$$

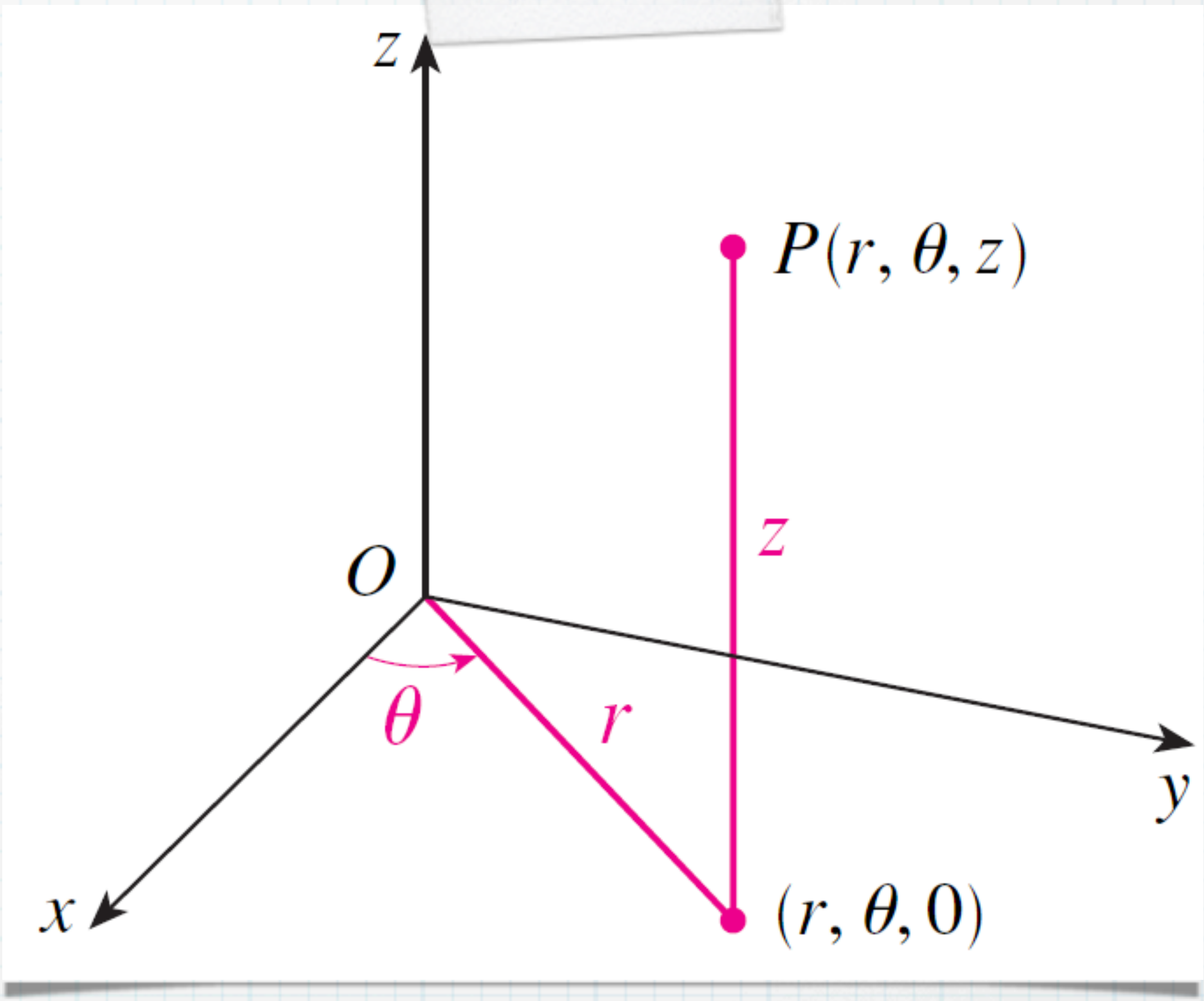
$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$



Cylindrical Coordinates



Cylindrical Coordinates



$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x} \qquad z = z$$

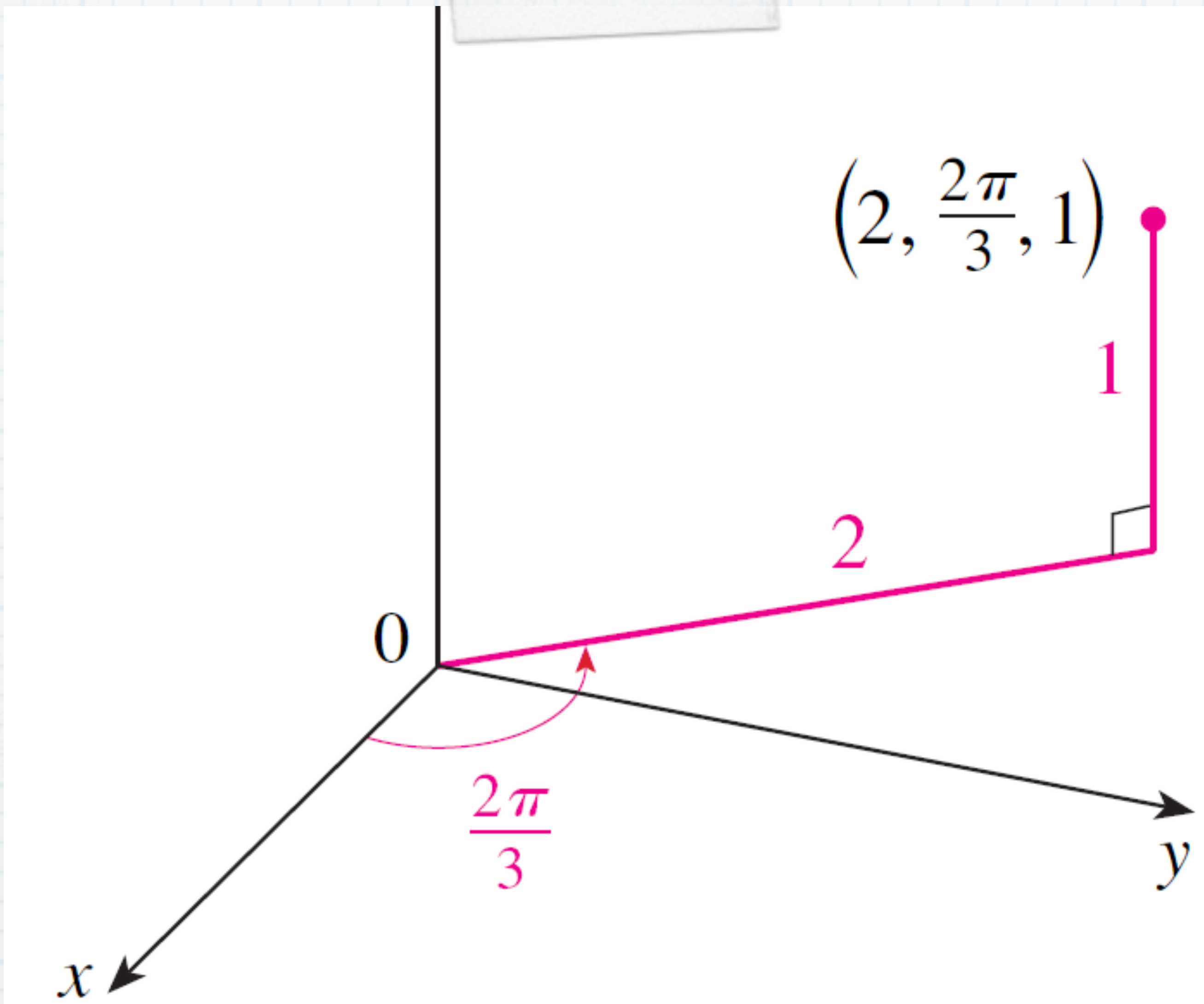
Converting

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

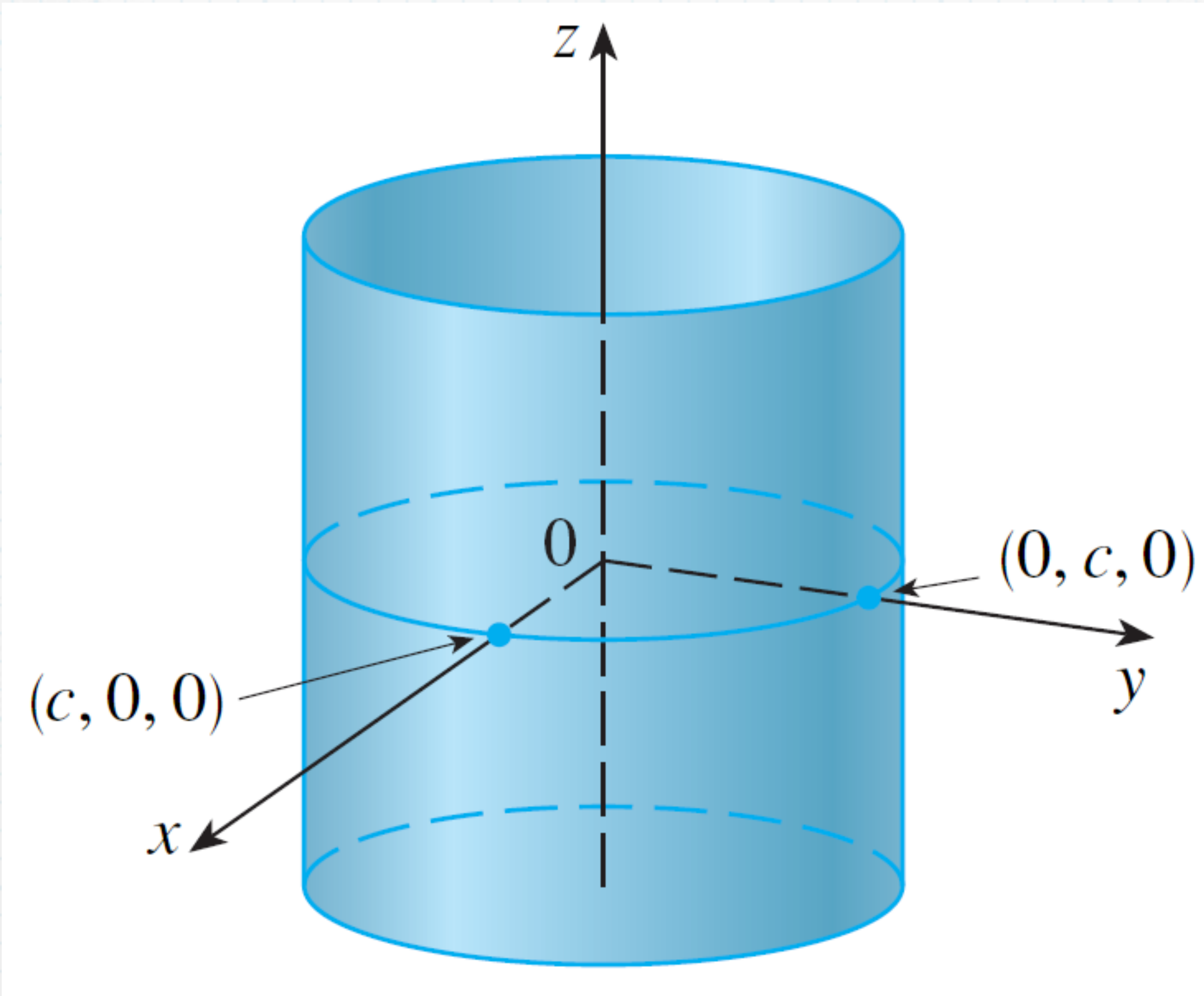
$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x} \qquad z = z$$

Cylindrical to Rectangular	Rectangular to Cylindrical
$x = r\cos\theta$	$r = \sqrt{x^2 + y^2}$
$y = r\sin\theta$	$\tan\theta = \dfrac{y}{x}$
$z = z$	$z = z$

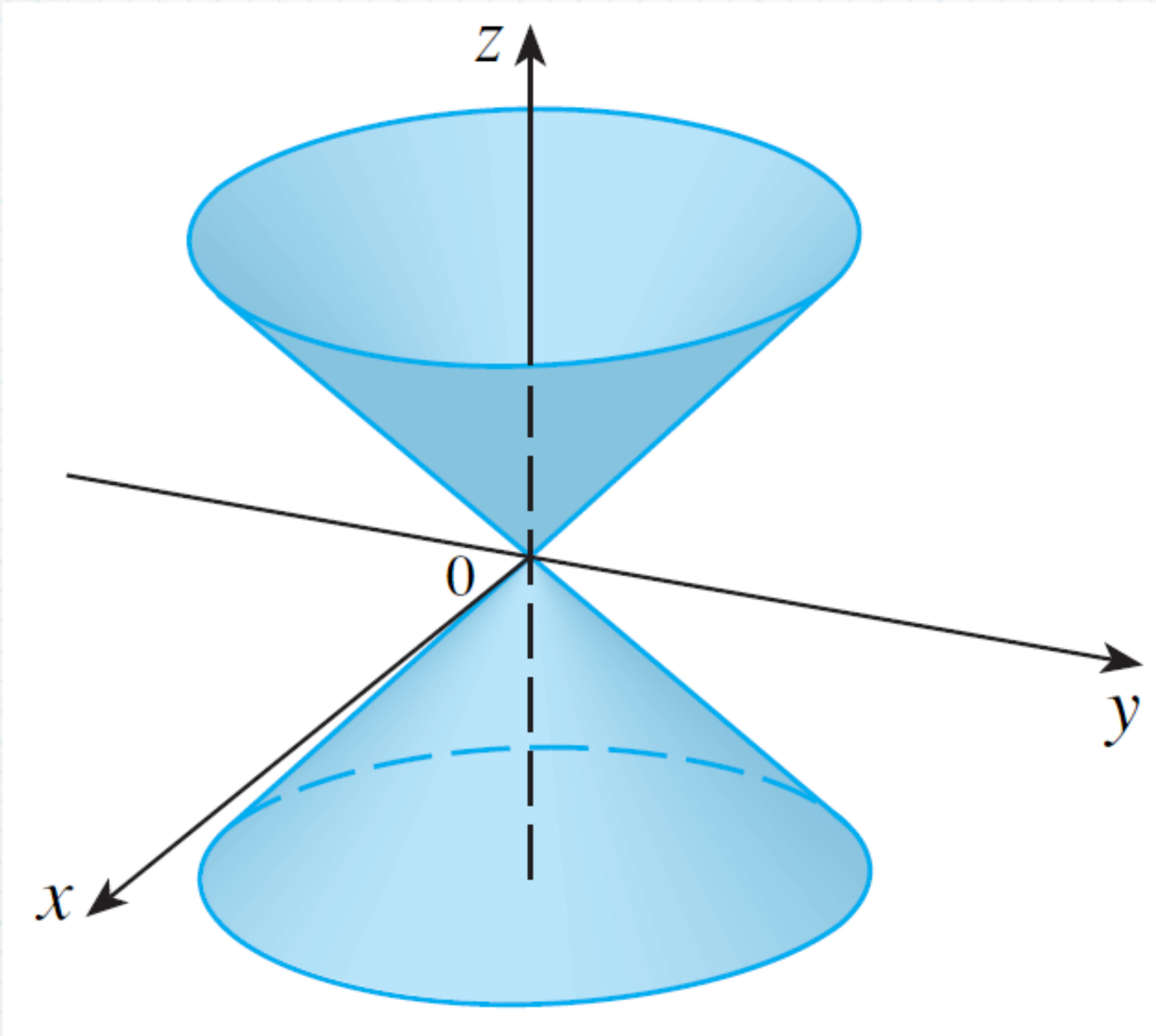
Example



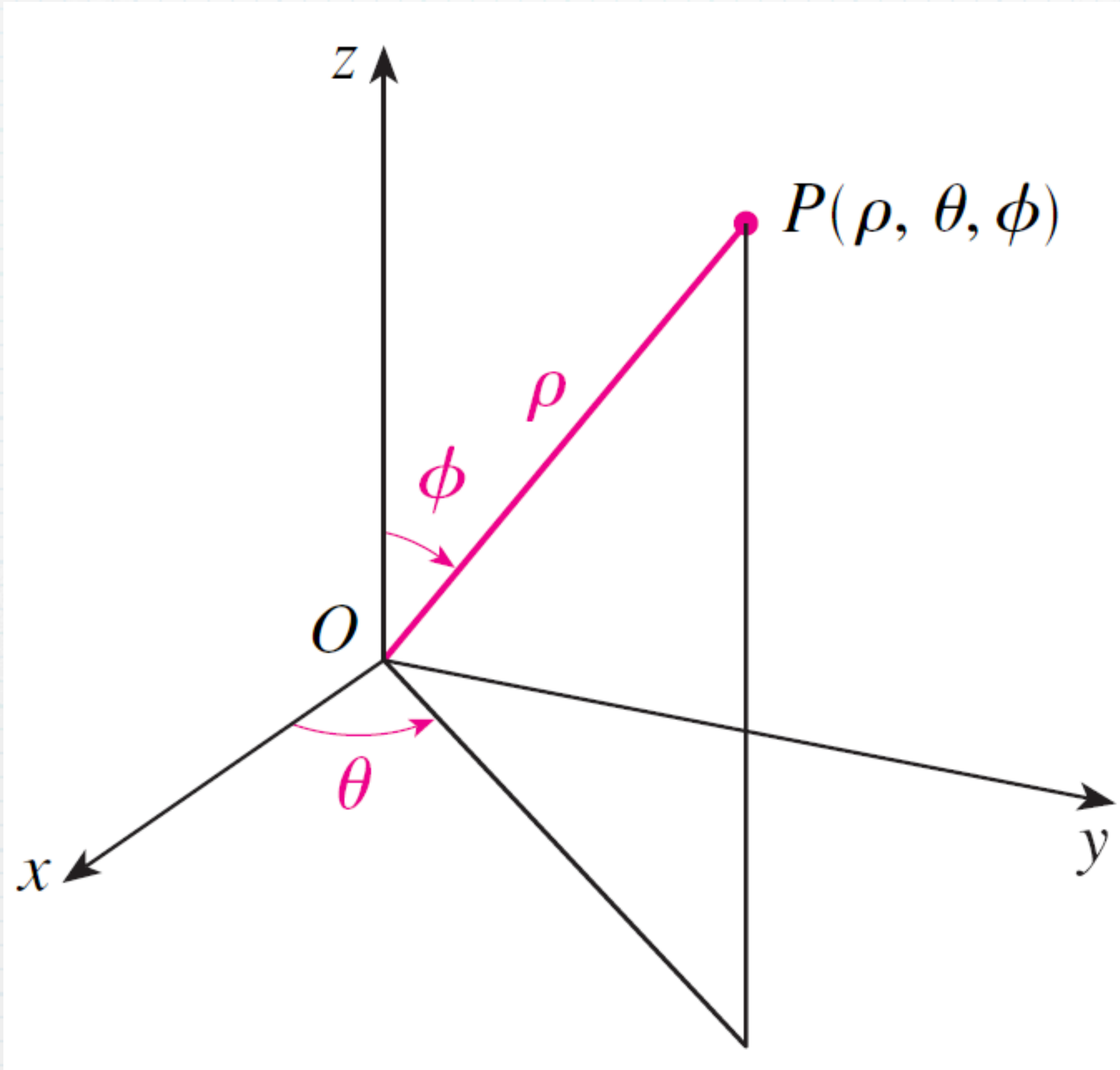
r=c



$z=r$



Spherical Coordinates



Spherical Coordinates

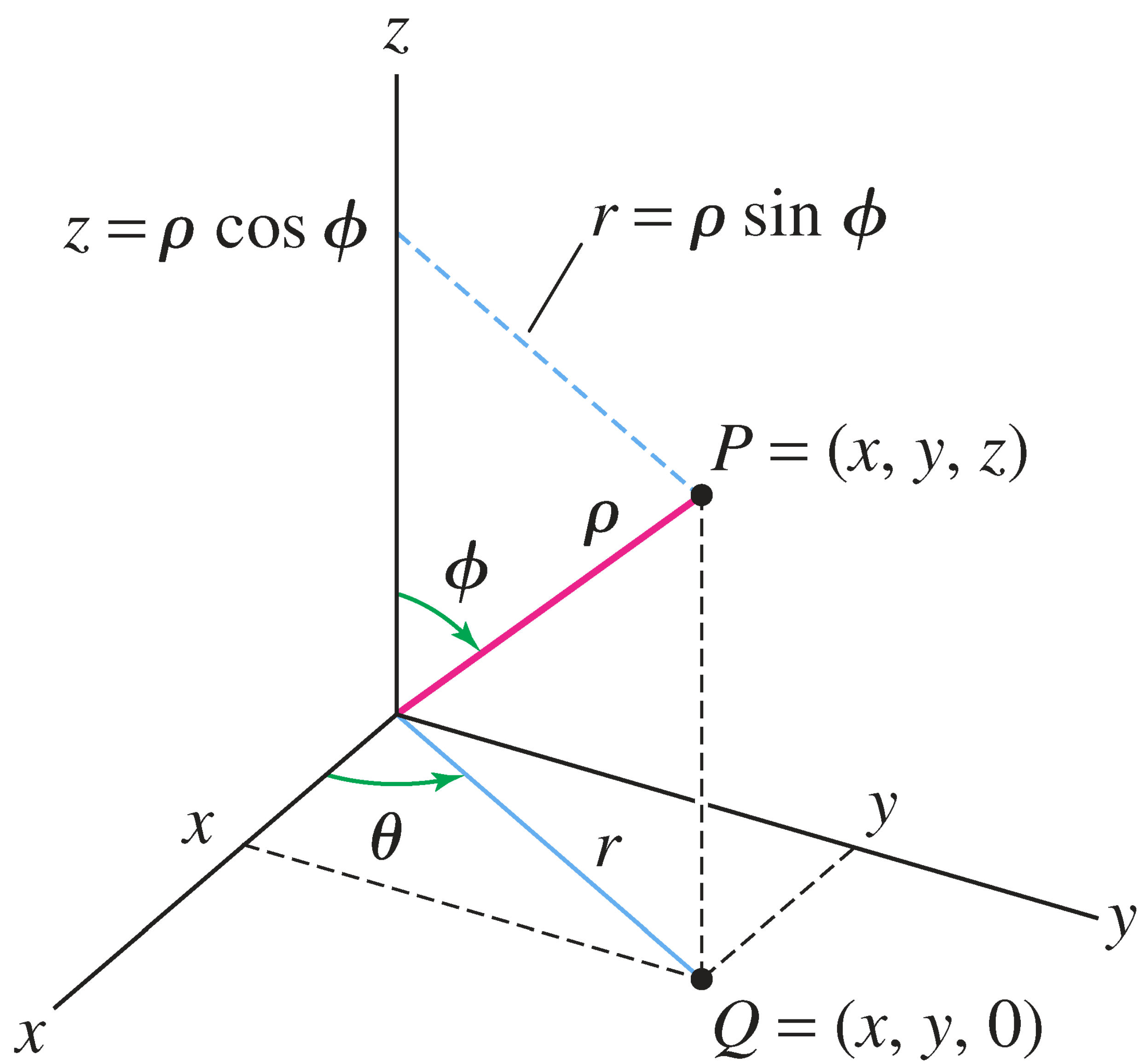


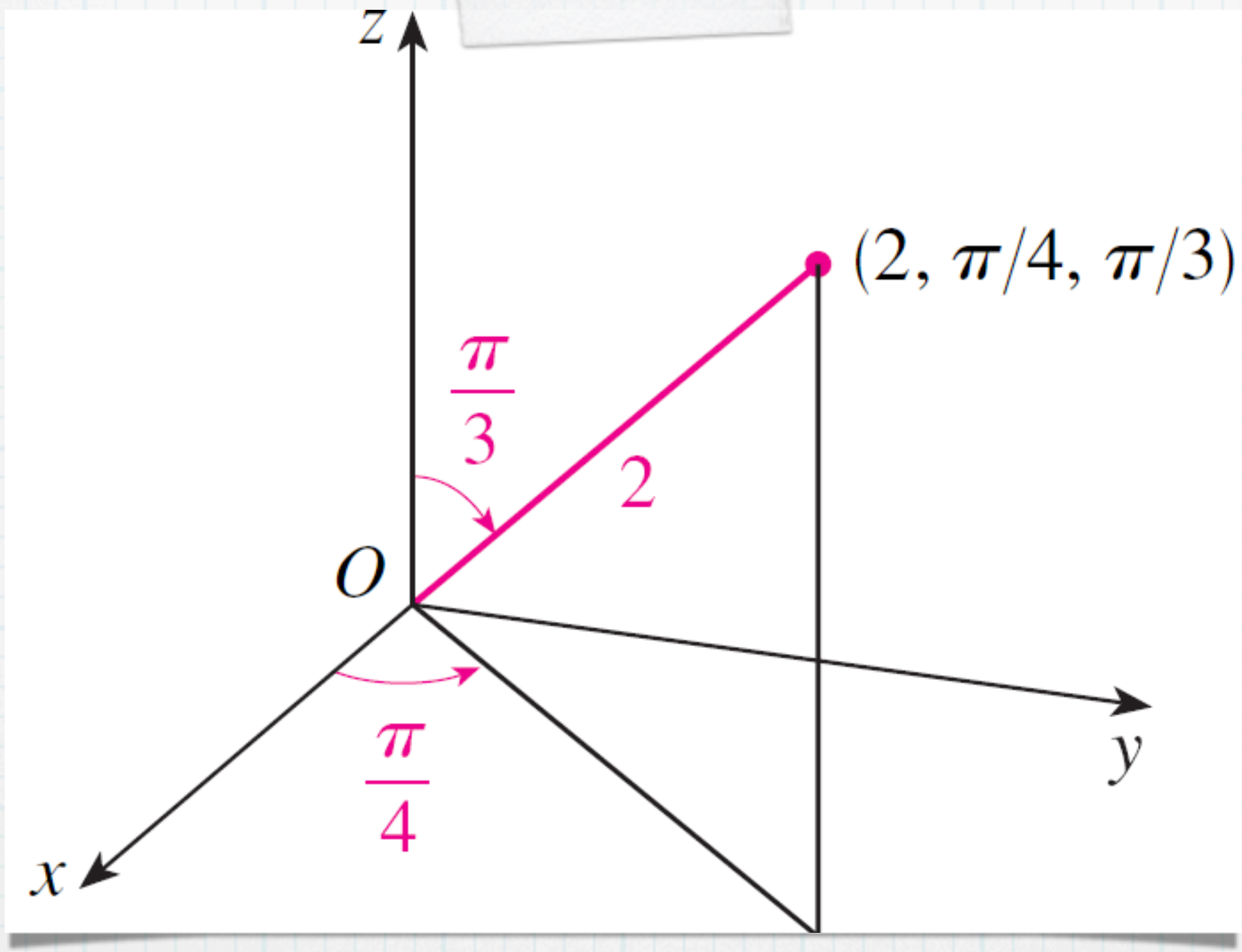
FIGURE 7 Use trigonometry to compute x, y, and z in terms of ρ, θ, and ϕ.

Converting

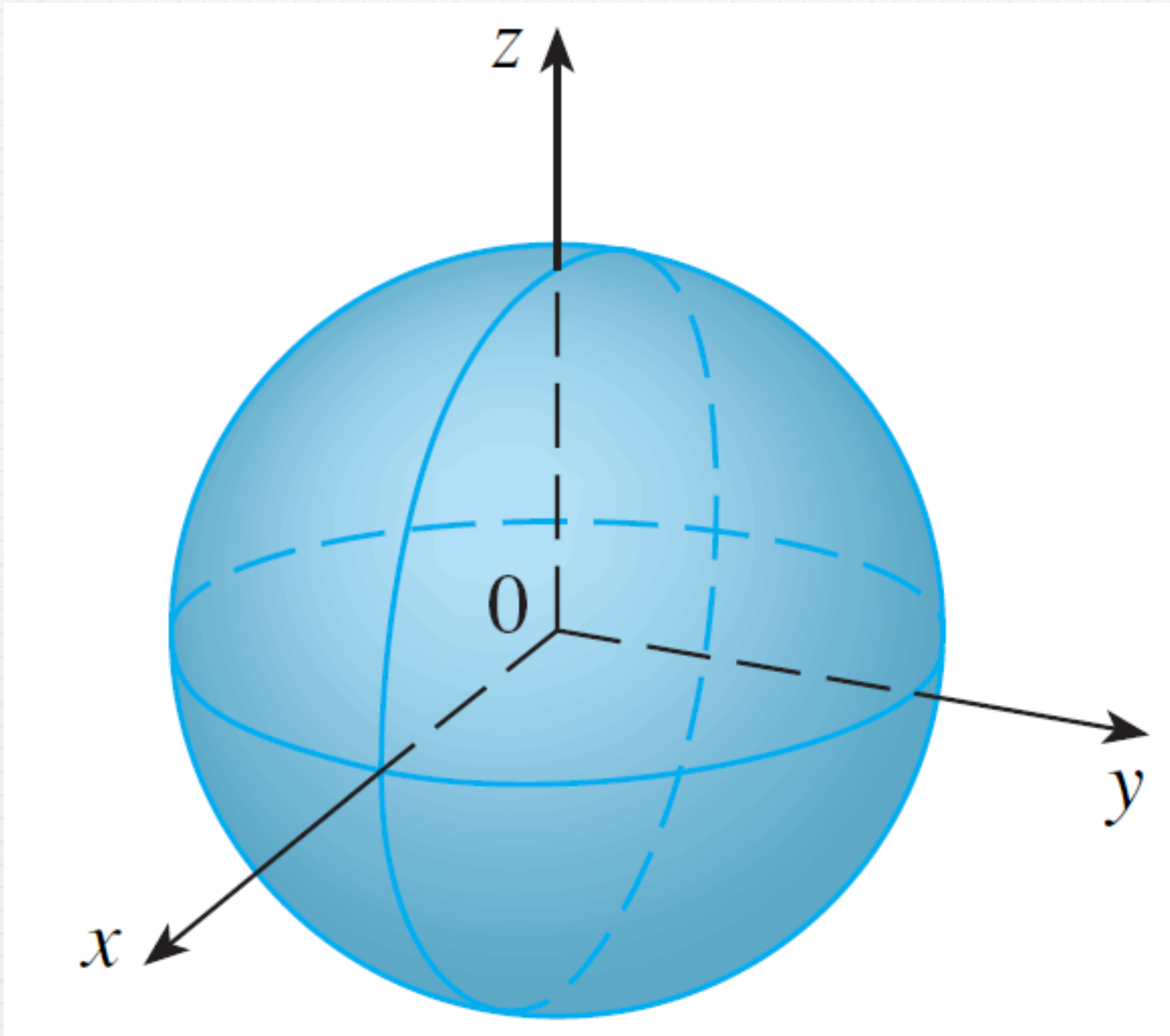
$$x = \rho \sin \phi \cos \theta \qquad y = \rho \sin \phi \sin \theta \qquad z = \rho \cos \phi$$

$$\rho^2 = x^2 + y^2 + z^2$$

Example

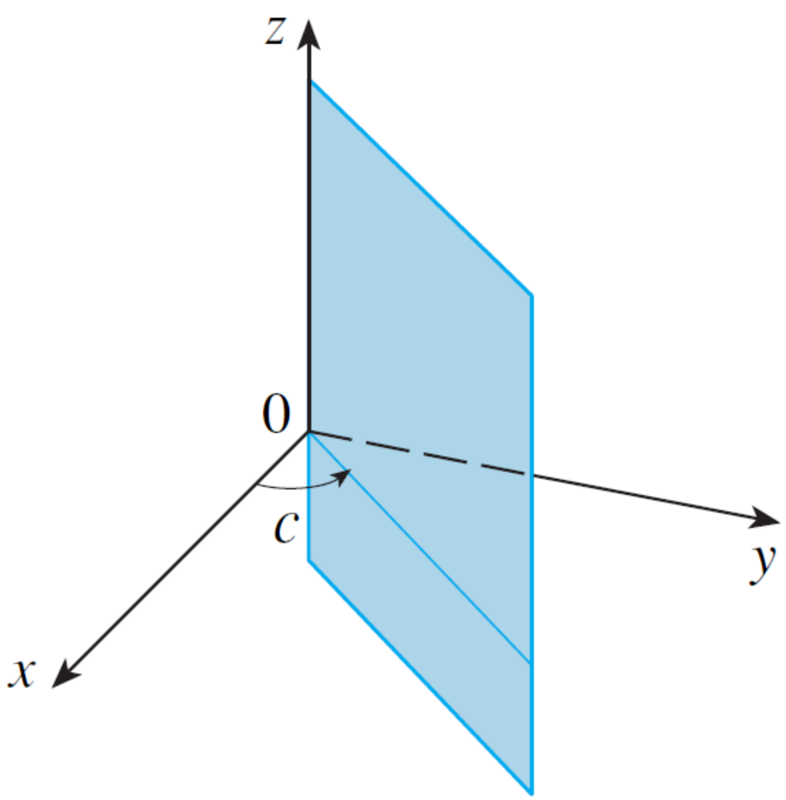


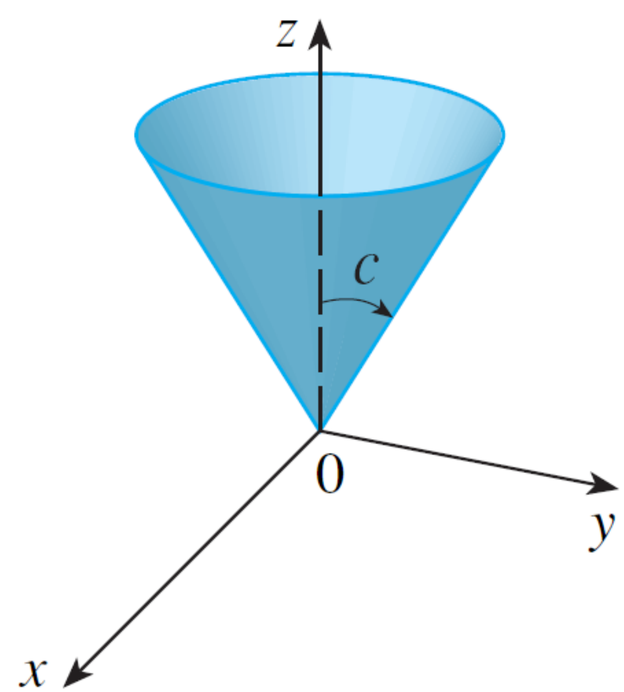
ρ=c



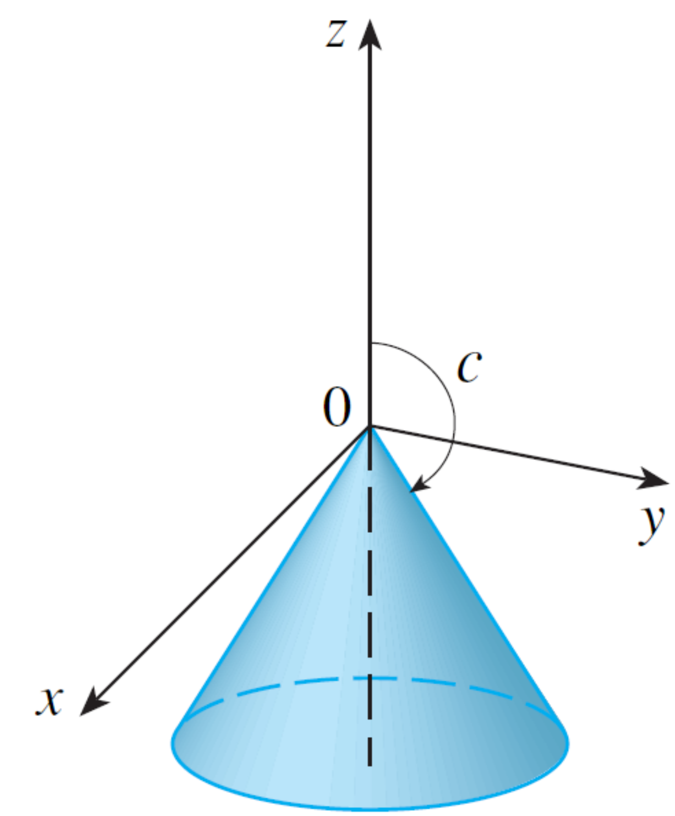
θ=c

φ=c





$0 < c < \pi/2$



$\pi/2 < c < \pi$

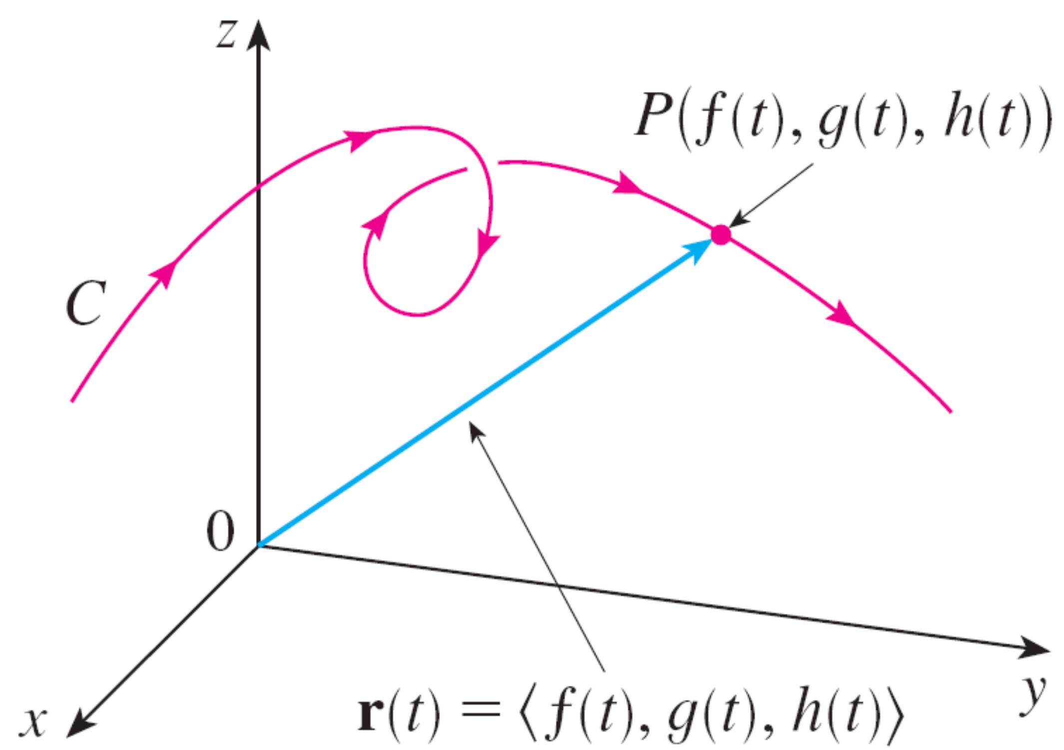
Activities

* Describe Me
* Mathematica: Coordinates

Section 10.1

Vector Functions and Space Curves

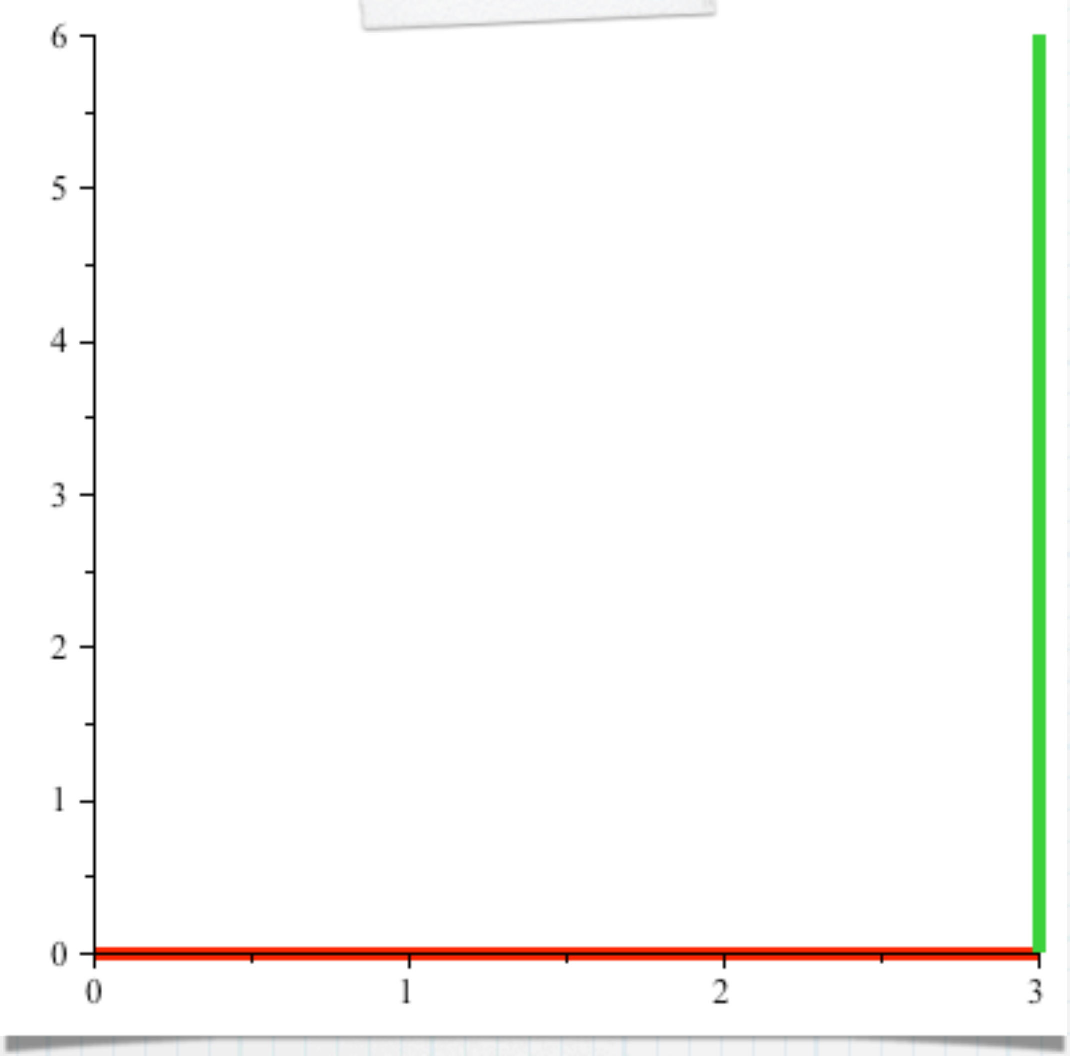
What is a Vector Function?

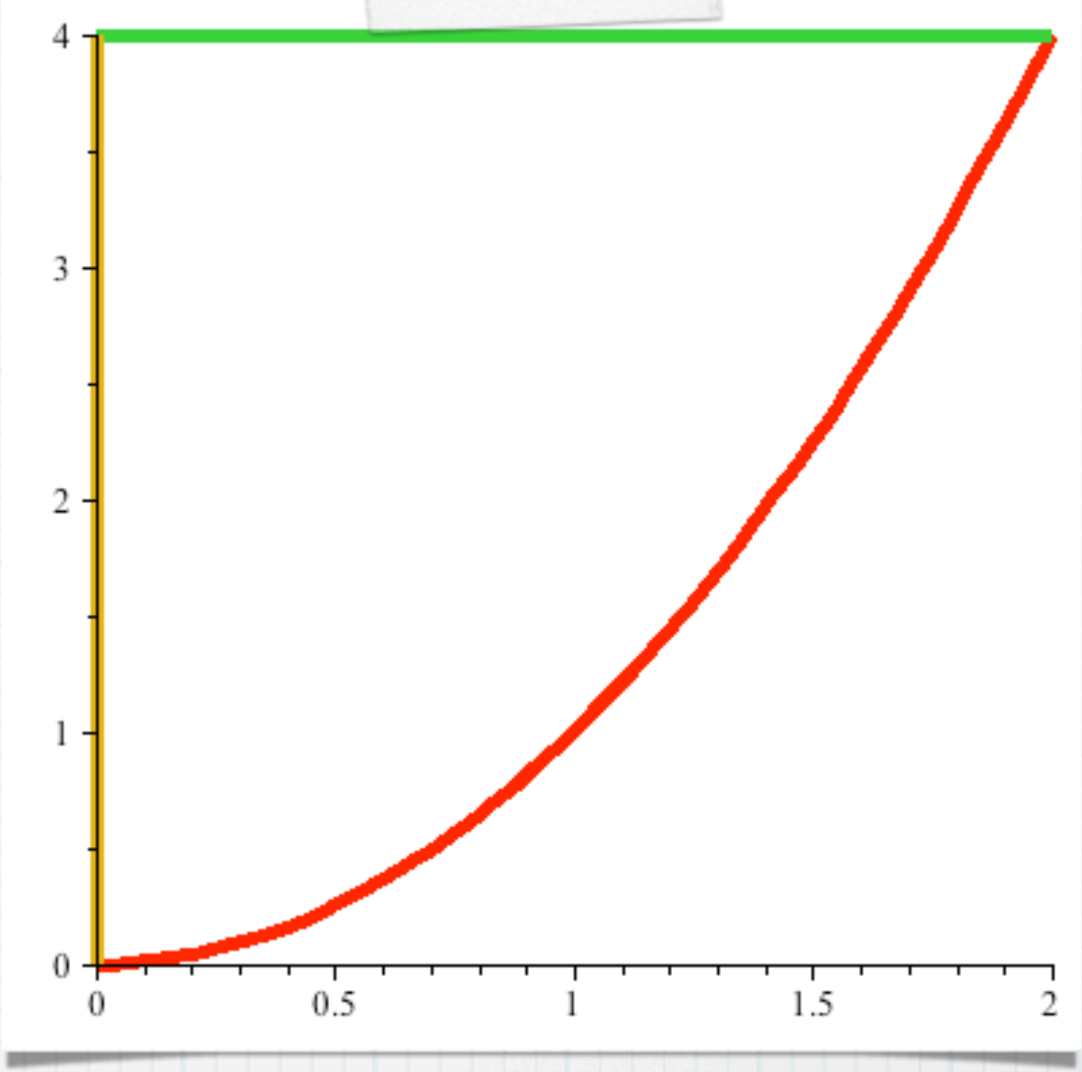


C is traced out by the tip of a moving position vector $\mathbf{r}(t)$.

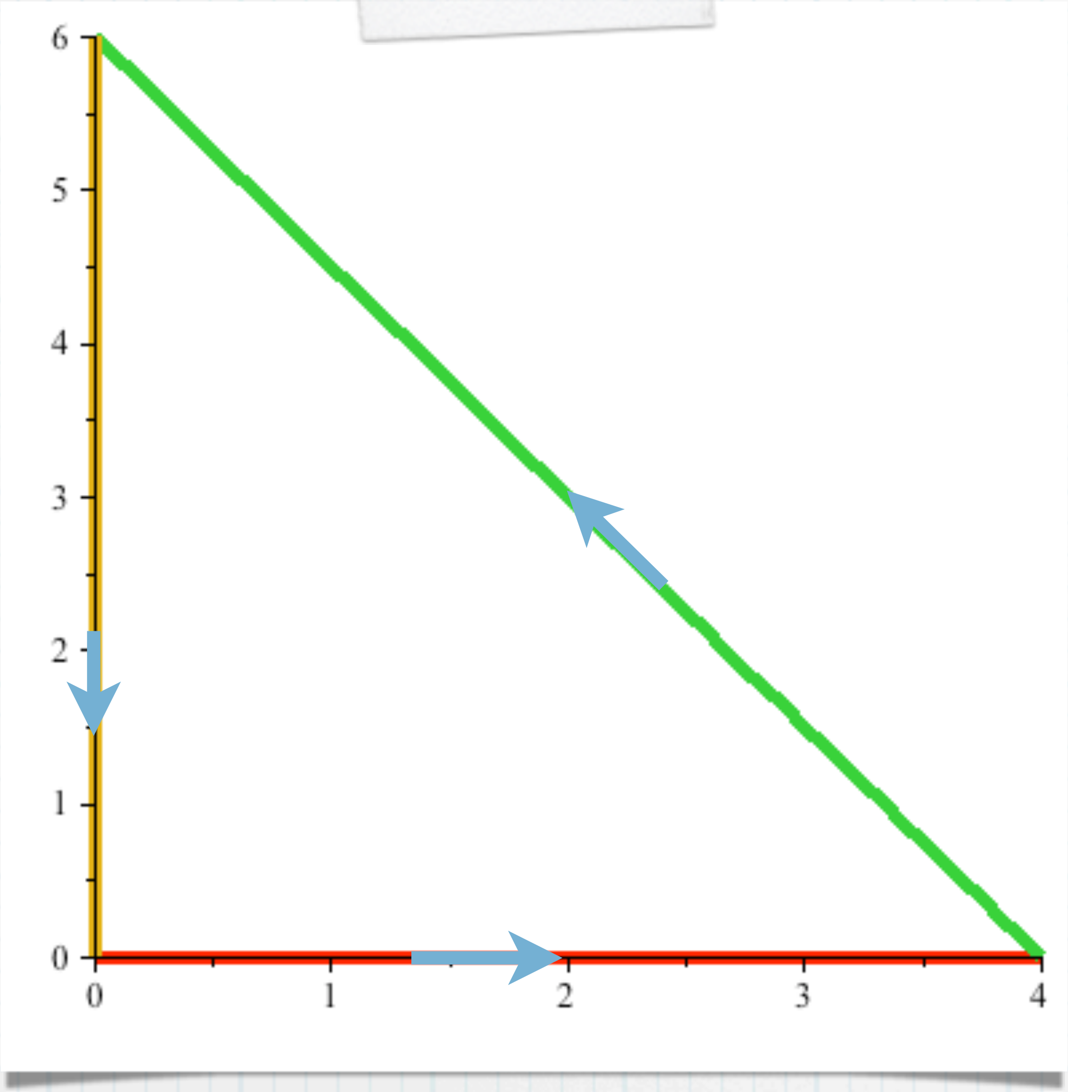
Vector Functions and Parametric Equations

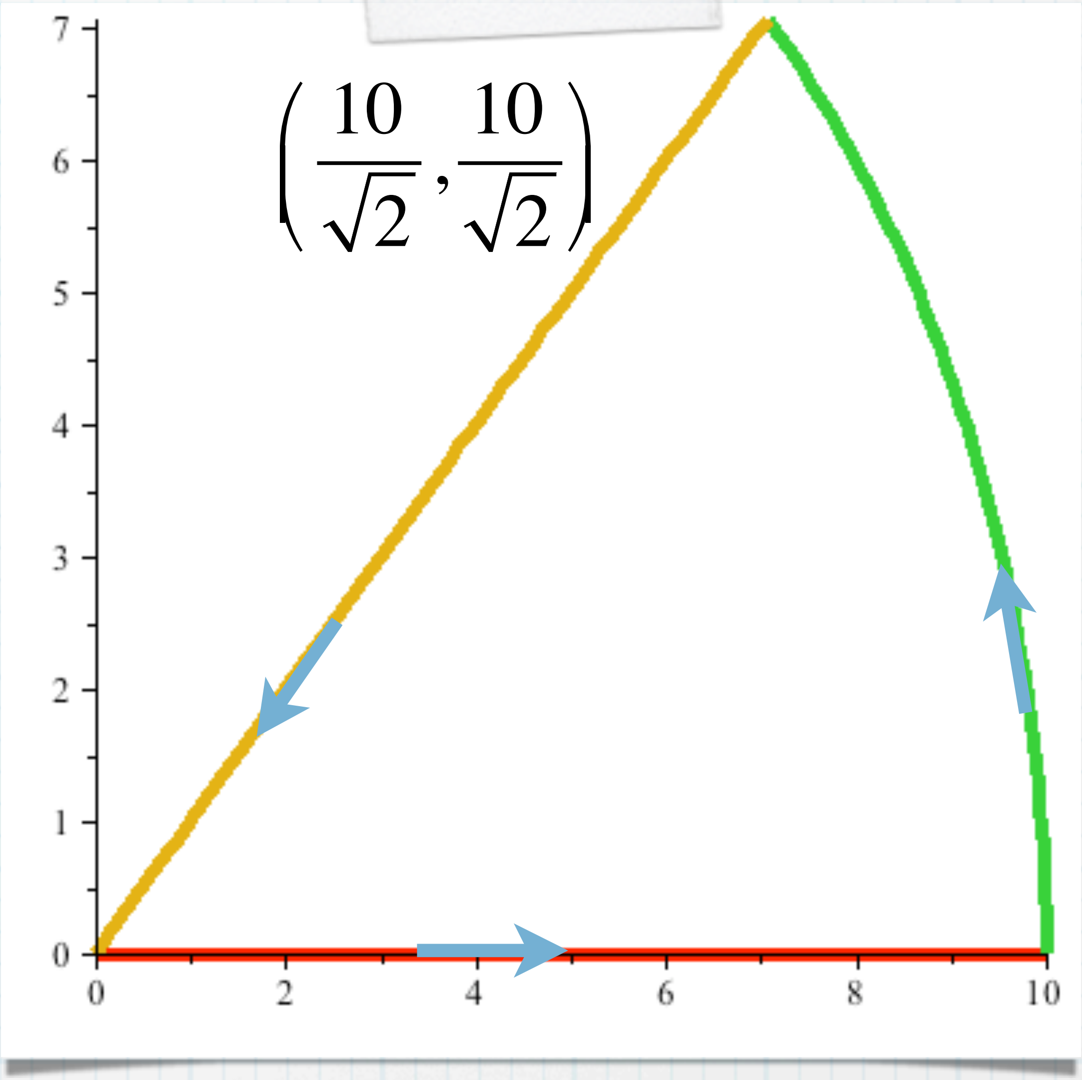
- $r(t)=\langle t,t^2\rangle$
- $x(t)=t$
  $y(t)=t^2$
- $y=x^2$
- Maple: Parametric Graphs
- plot([x(t),y(t),t=a..b])



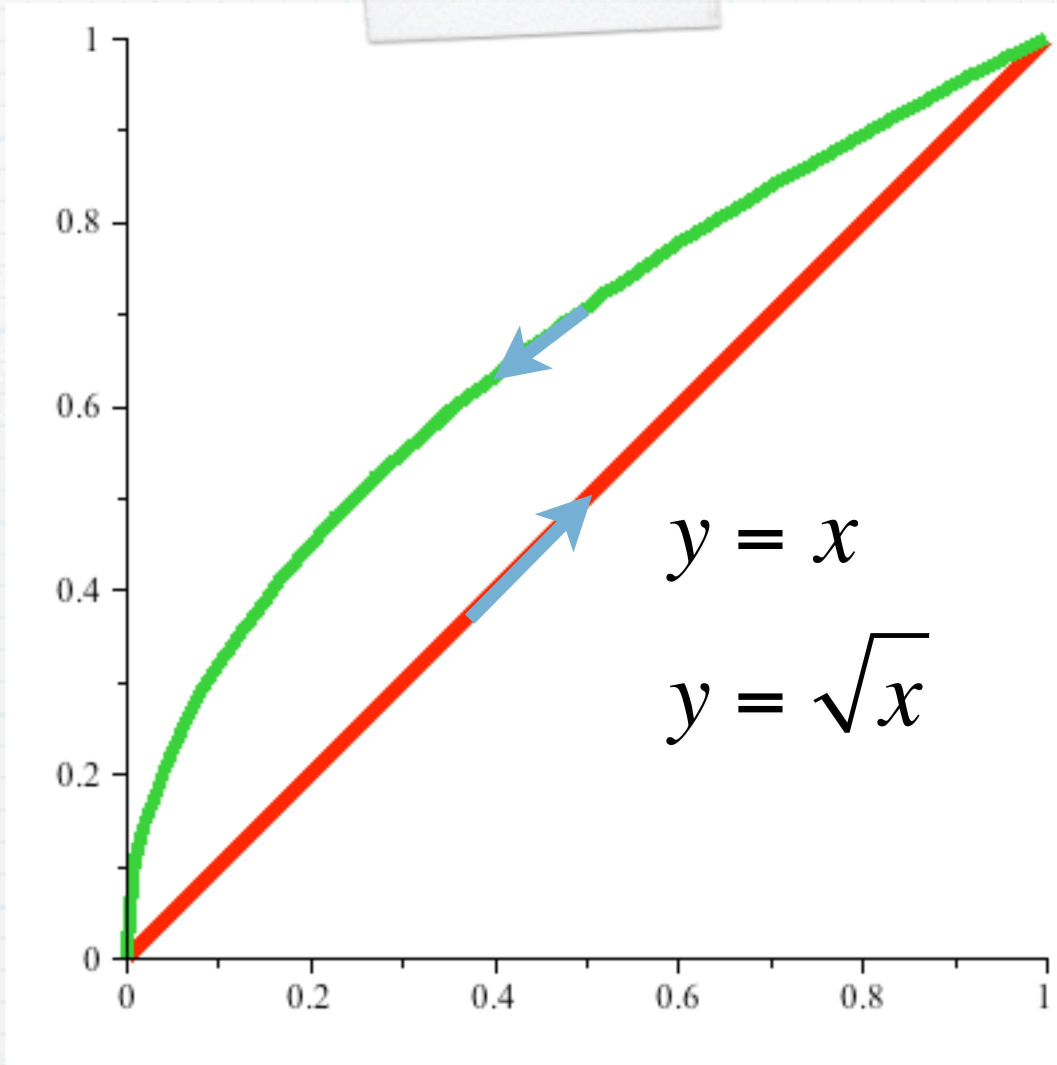


Examples





Problems

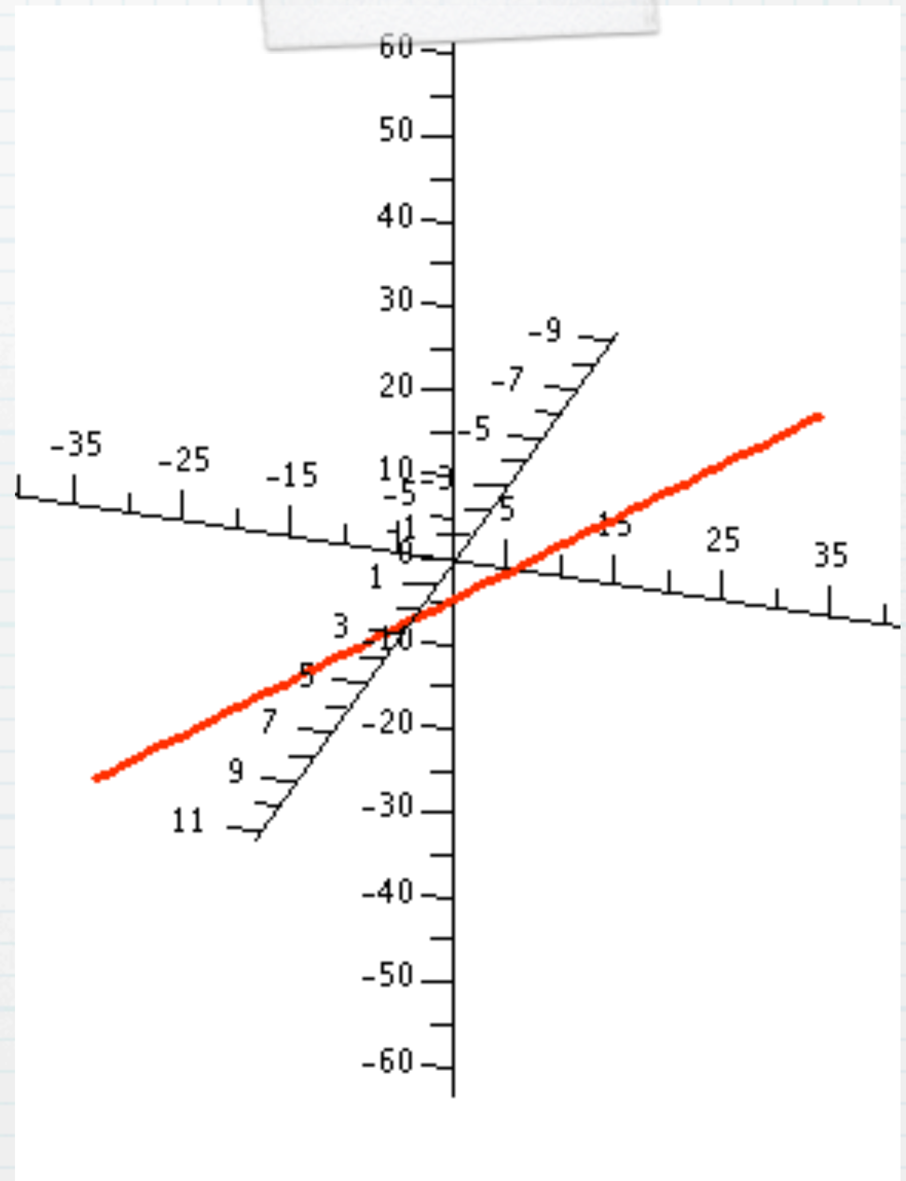


Problems

3D Example

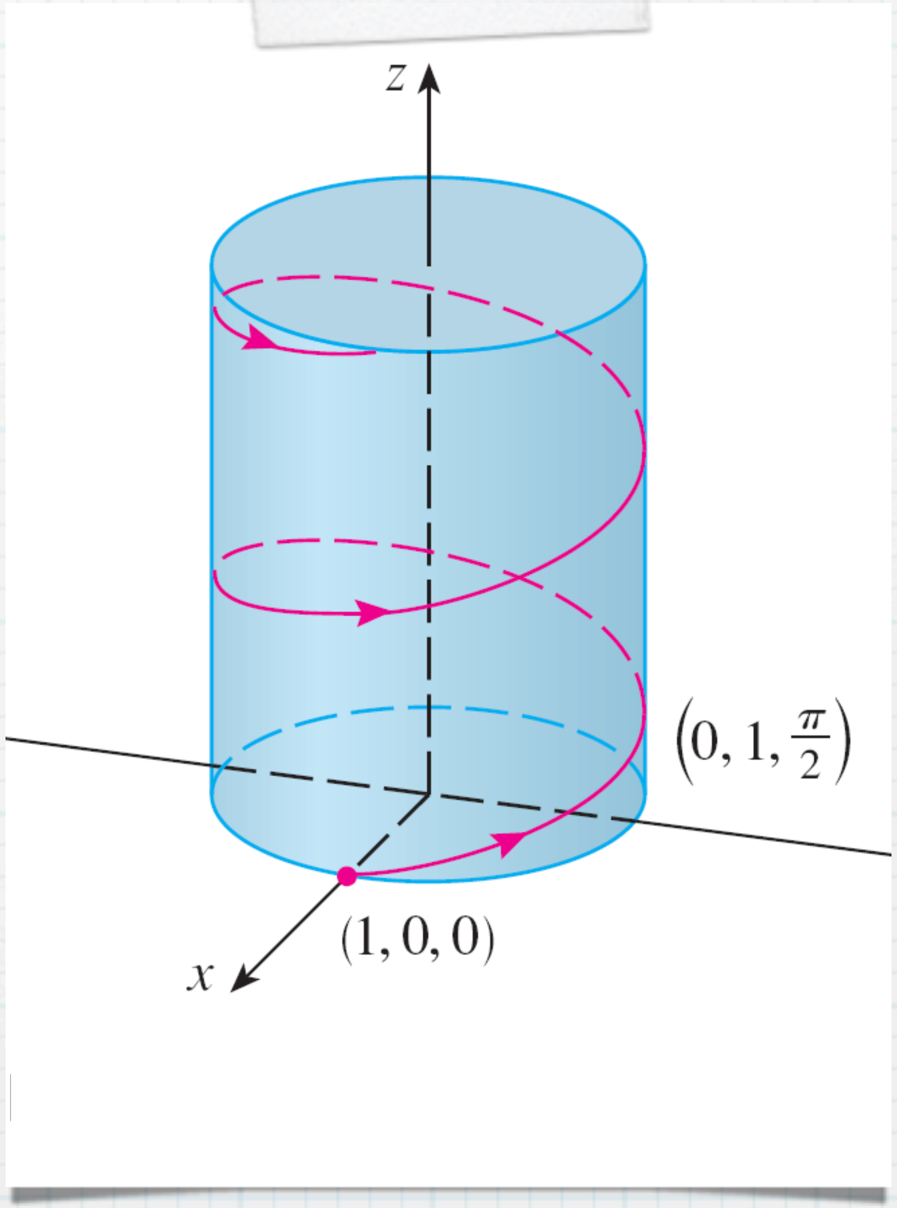
r(t)=<1+t, 2+5t, -1+6t>

* x=1+t
* y=2+5t
* z=-1+6t
* Line
    * direction <1,5,6>
    * through point P(1,2,-1)



Using Surfaces to Graph Vector Functions

* $r(t) = \langle \cos(t), \sin(t), t \rangle$
* lies on $x^2 + y^2 = 1$



Projections

* r(t)=<cos(t),sin(t),t>

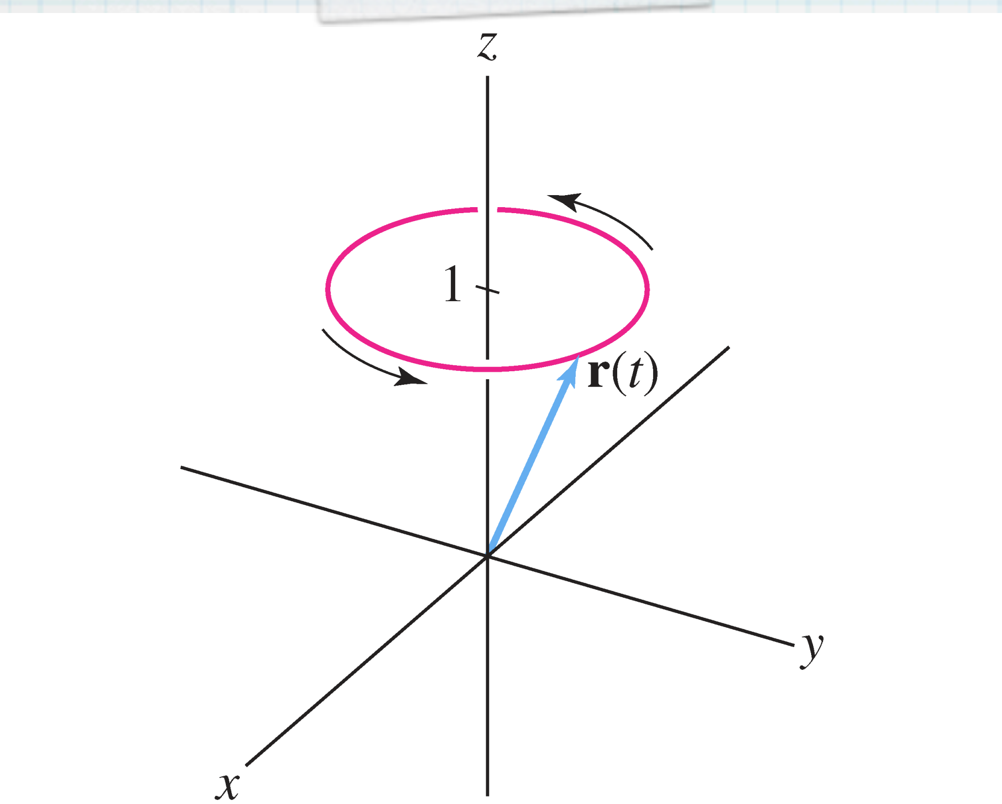


FIGURE 3 Plot of $\mathbf{r}(t) = \langle \cos t, \sin t, 1 \rangle$.

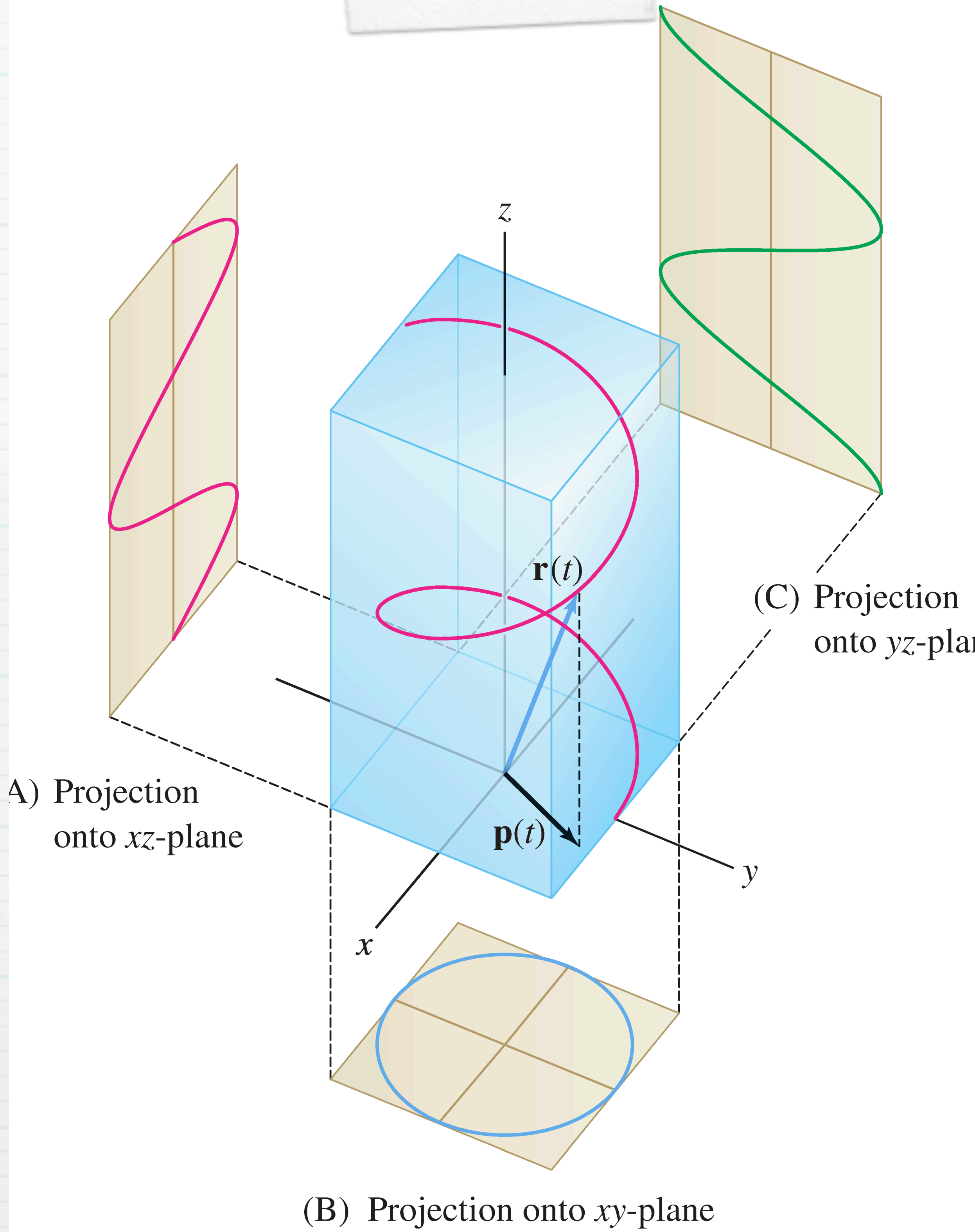
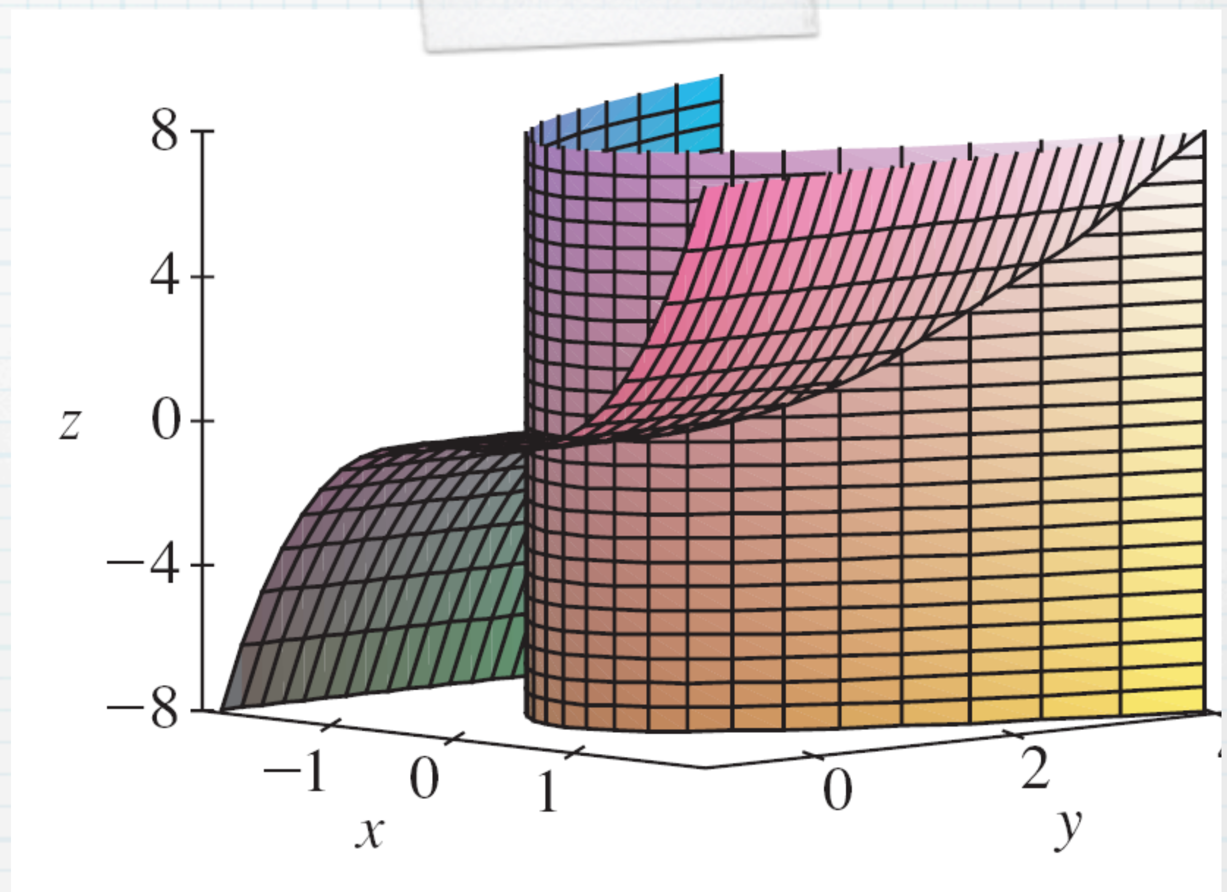


FIGURE 2 Projections of the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$.

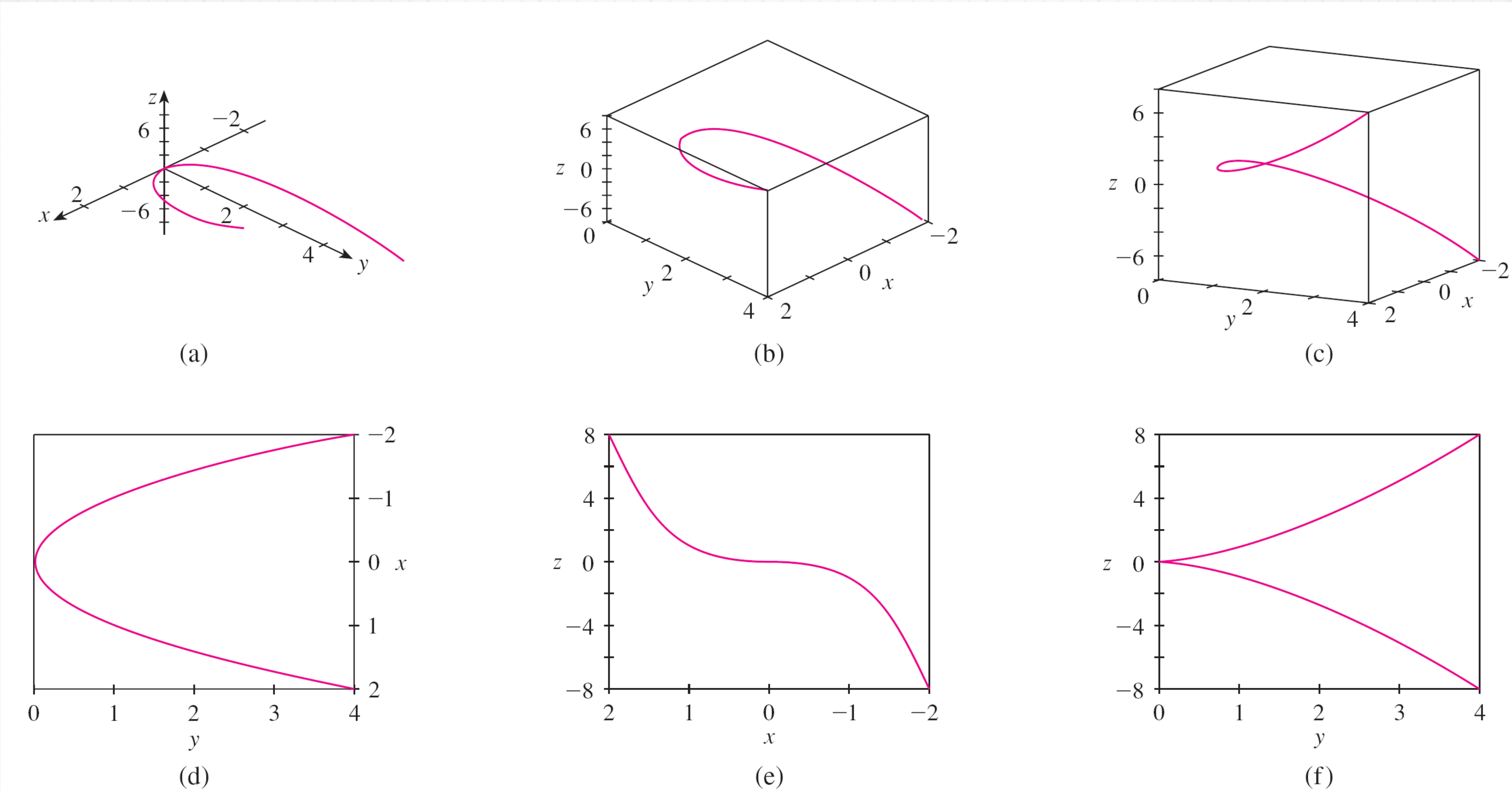
Using Surfaces to Graph Vector Functions

- $r(t)=\langle t,t^2,t^3\rangle$
- cylinder: $y=x^2$
- cylinder: $z=x^3$
- Intersection of the two cylinders
- Show Maple



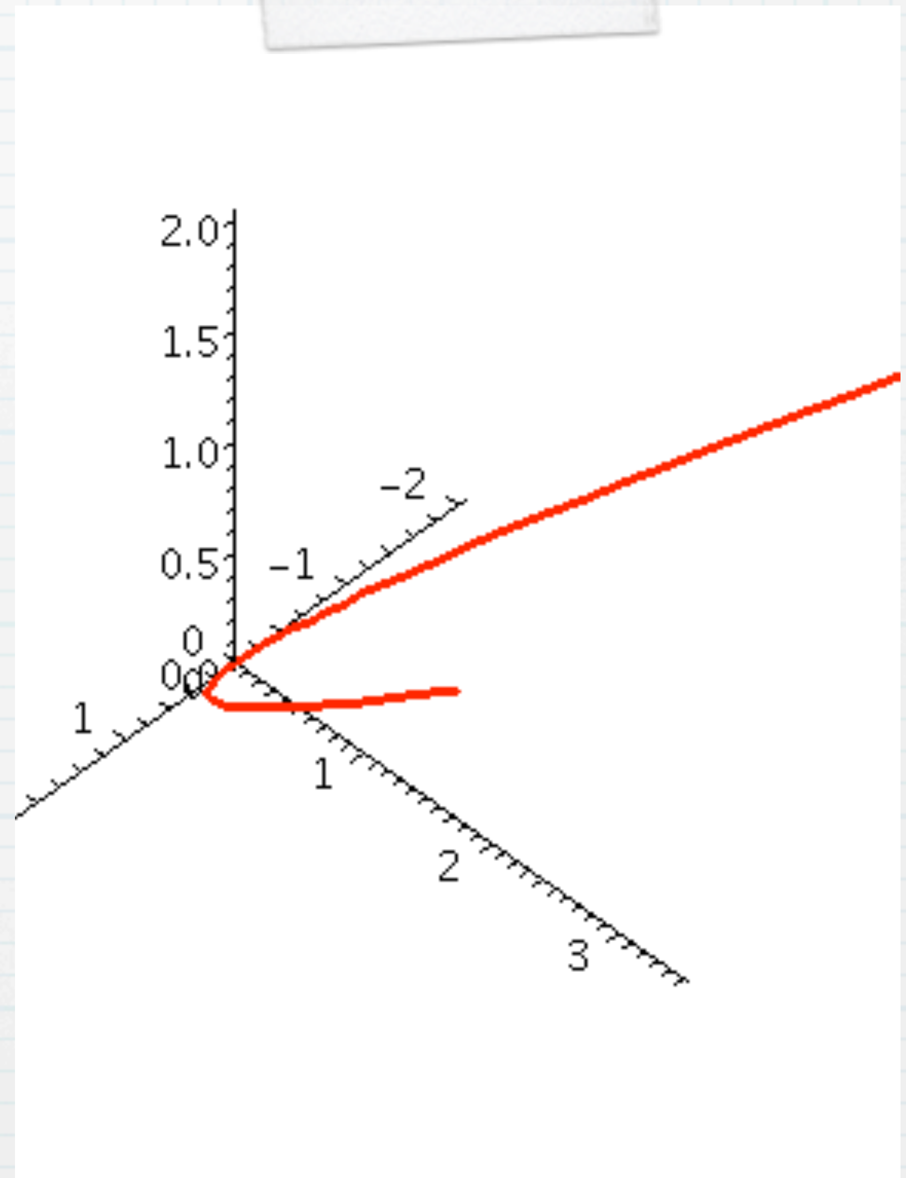
Traces

$r(t)=\langle t, t^2, t^3\rangle$



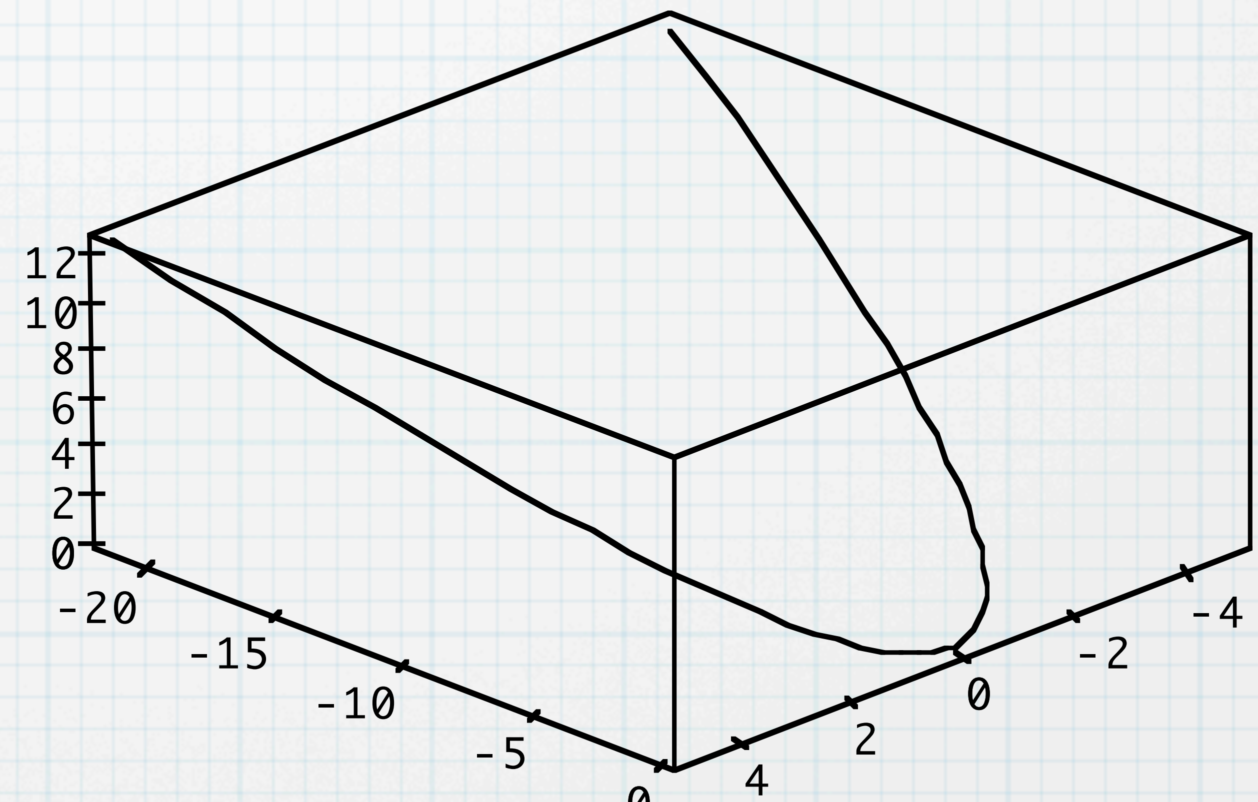
Traces and Projections

* Handout: Projections



Spacecurve Problems

* Mathematica: Space Curves and Surfaces

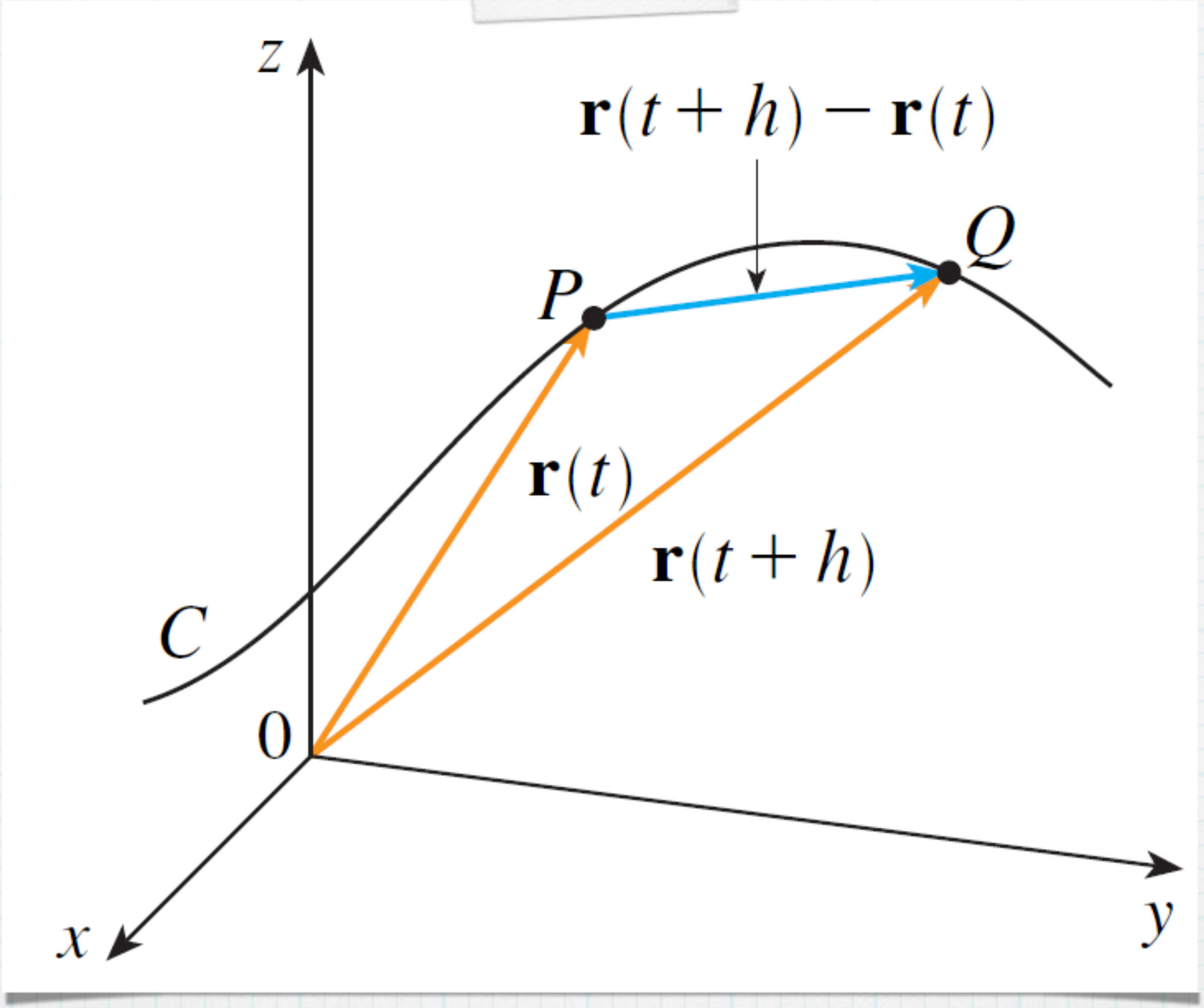


Section 10.2

Derivatives and Integrals of Vector Functions

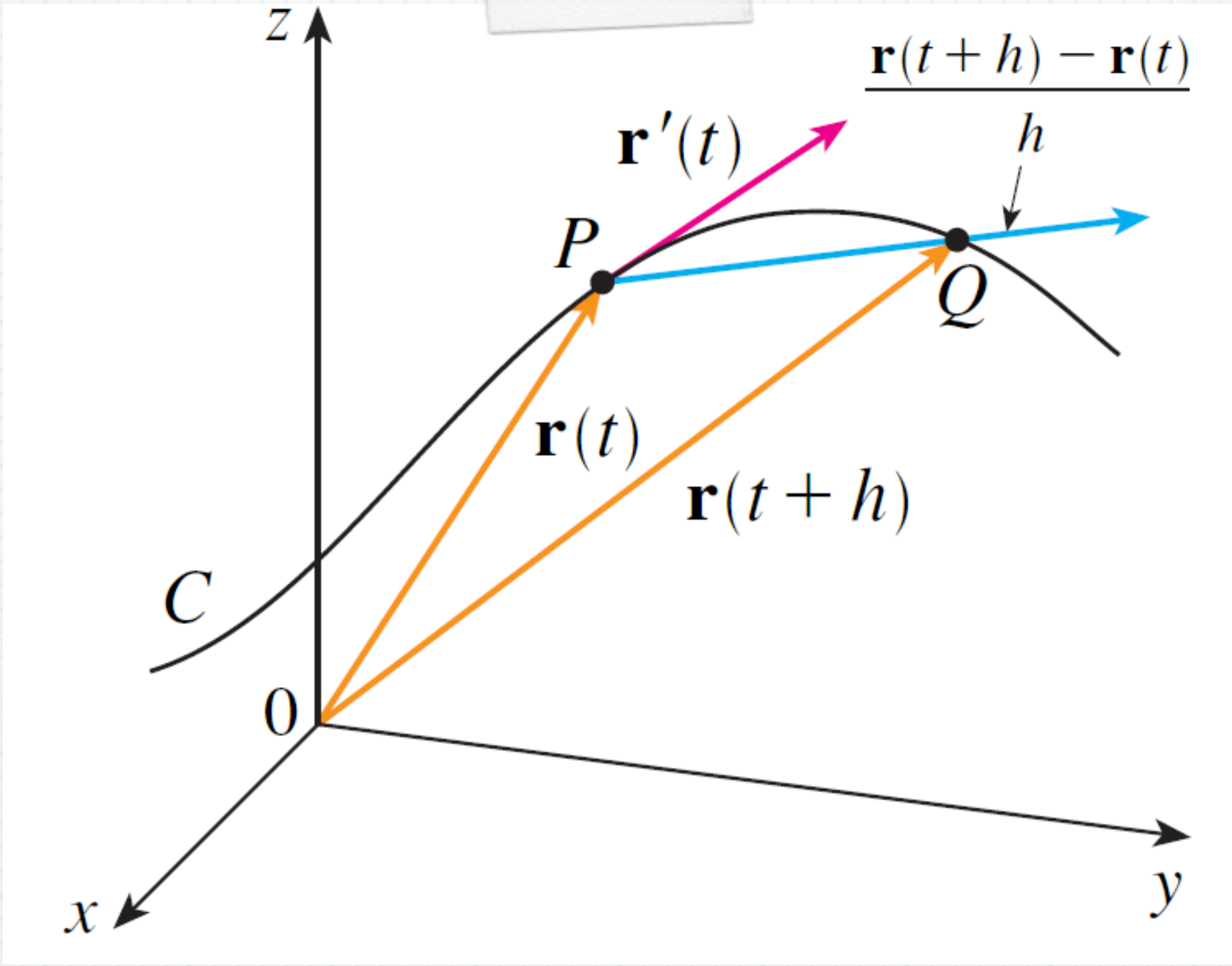
Derivatives

$$\frac{d\mathbf{r}}{dt} = \mathbf{r}'(t) = \lim_{h \to 0} \frac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h}$$



Geometry

Tangent Vector



$$T(t) = \frac{r'(t)}{\left| r'(t) \right|}$$

Theorem

$\boxed{2}$ **Theorem** If $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\,\mathbf{i} + g(t)\,\mathbf{j} + h(t)\,\mathbf{k}$, where f, g, and h are differentiable functions, then

$$\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle = f'(t)\,\mathbf{i} + g'(t)\,\mathbf{j} + h'(t)\,\mathbf{k}$$

Differentiation Rules

3 Theorem Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, c is a scalar, and f is a real-valued function. Then

1. $\dfrac{d}{dt}[\mathbf{u}(t) + \mathbf{v}(t)] = \mathbf{u}'(t) + \mathbf{v}'(t)$

2. $\dfrac{d}{dt}[c\mathbf{u}(t)] = c\mathbf{u}'(t)$

3. $\dfrac{d}{dt}[f(t)\mathbf{u}(t)] = f'(t)\mathbf{u}(t) + f(t)\mathbf{u}'(t)$

4. $\dfrac{d}{dt}[\mathbf{u}(t) \cdot \mathbf{v}(t)] = \mathbf{u}'(t) \cdot \mathbf{v}(t) + \mathbf{u}(t) \cdot \mathbf{v}'(t)$

5. $\dfrac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$

6. $\dfrac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u}'(f(t))$ (Chain Rule)

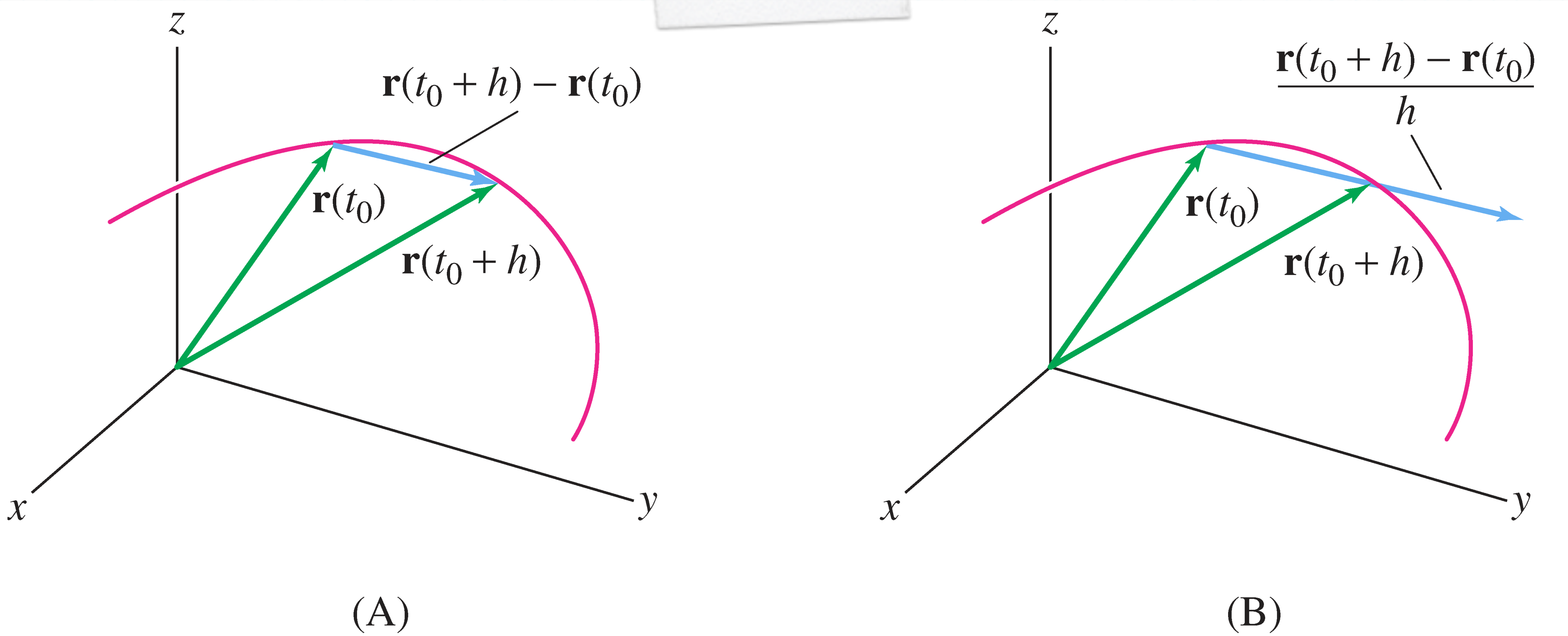
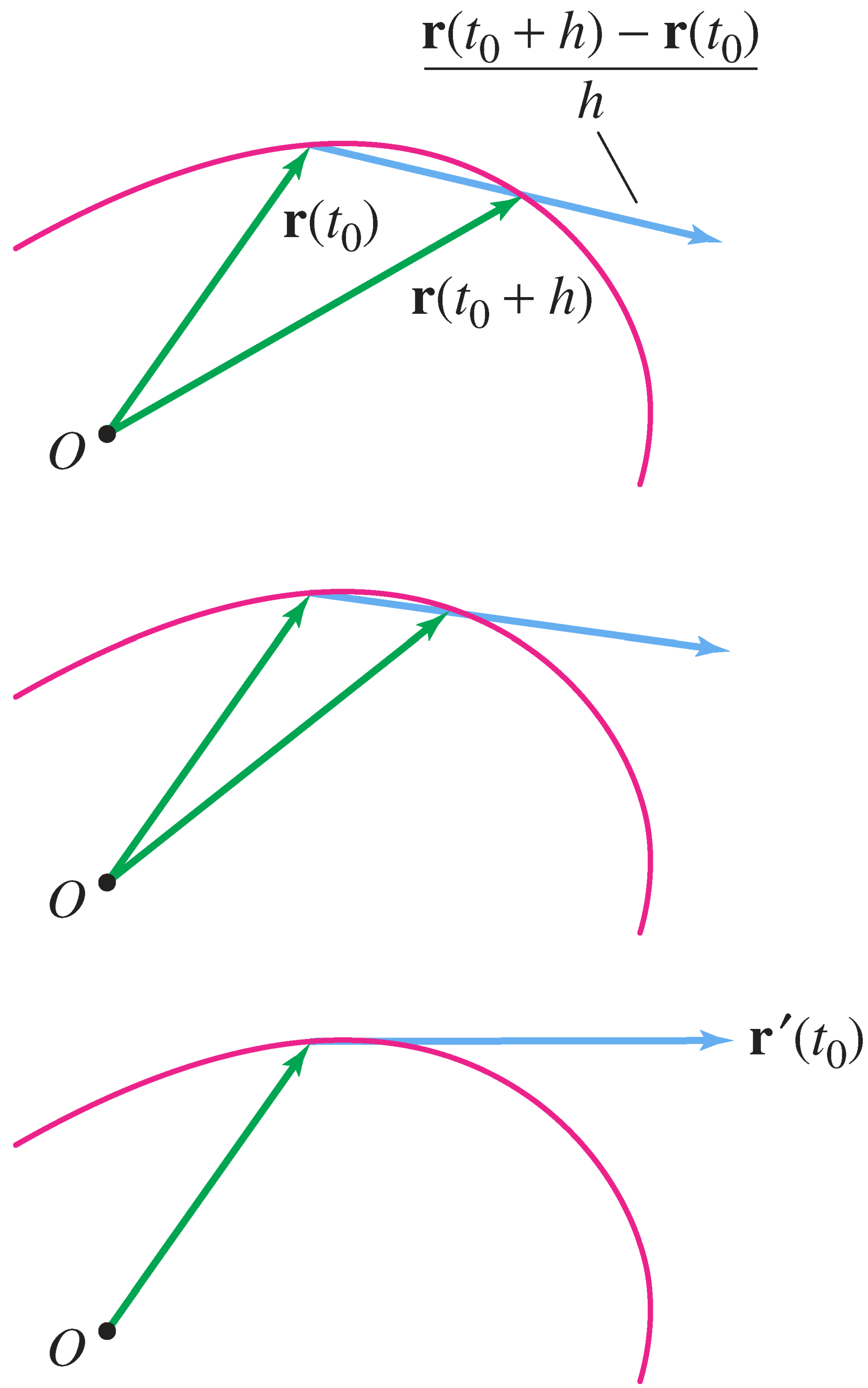


FIGURE 2 The difference quotient points in the direction of $\Delta\mathbf{r} = \mathbf{r}(t_0 + h) - \mathbf{r}(t_0)$.

Tangent Vector



Vector pointing in direction tangent to the curve at the point to

FIGURE 3 The difference quotient converges to a vector $\mathbf{r}'(t)$, tangent to the curve $\mathbf{r}(t)$.

$$\boxed{\text{Tangent line at } \mathbf{r}(t_0): \qquad \mathbf{L}(t) = \mathbf{r}(t_0) + t\,\mathbf{r}'(t_0)}$$

Theorem

* If a vector function has constant length, then its derivative is perpendicular to the vector
* Example: $r(t)=\langle\cos(t),\sin(t)\rangle$

Unit Tangent

* The Unit Tangent has constant length 1.
* The only quantity that changes over time is its direction.
* Abs(dT/dt) measures the rate of change of the direction of the unit tangent vector.
* Example: $r(t)=\langle t^3, t^6 \rangle$

Integrals

$$\int_a^b \mathbf{r}(t)\,dt = \left(\int_a^b f(t)\,dt\right)\mathbf{i} + \left(\int_a^b g(t)\,dt\right)\mathbf{j} + \left(\int_a^b h(t)\,dt\right)\mathbf{k}$$

$$\boxed{\int_a^b \mathbf{r}(t)\,dt = \mathbf{R}(t)\Big]_a^b = \mathbf{R}(b) - \mathbf{R}(a)}$$